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Constraints on Brane and Bulk Ideal Fluid in Randall-Sundrum Cosmologies

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Abstract

We investigate constraints for including bulk and brane matter in the Randall-Sundrum model. In the static configuration with two zero thickness branes, we find that *no* realistic brane matter is possible when the radion is stabilized. We also consider the possibility that the radion has stabilized by dissipating its energy into the bulk in the form of some unspecified matter, and find the Randall-Sundrum cosmological solutions in the presence of bulk ideal fluid. We discover that there is only one allowed equation of state, $p = \rho$, corresponding to the stiff ideal fluid. We find the corresponding brane cosmologies and compare them with the Friedmann-Robertson-Walker model.

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1 Introduction

Randall and Sundrum have recently suggested a novel solution to the hierarchy problem involving extra dimensions [1, 2], which has attracted much attention. The RS proposal assumes a five-dimensional spacetime where the extra dimension is a S^1/Z_2 orbifold, with branes located at the two orbifold fixed points. The branes are thus at the spatial boundaries of the bulk spacetime. The brane tensions together with a negative bulk vacuum energy provide the source for the 5-d Einstein equation, which yields as the solution the metric

$$ds^2 = e^{-2k|y|}\ (-dt^2 + \sum_{i=1}^{3}(dx^i)^2) + dy^2\ . \tag{1}$$

The exponential warp factor rescales the physical masses on the second "TeV brane", giving rise to a large suppression factor with respect to the Planck mass, and hence a possible solution to the hierarchy problem. However, this is achieved at the expense of fine tuning: the cosmological constants on the two branes must be of equal magnitude but with opposite signs. The size of the extra dimension, or the radion, is a four dimensional modulus field whose value needs to be fixed at the right scale in order for the RS scenario to work.

The RS model has been refined in many ways since its inception, but the question of radion stabilization remains unsolved. According to commonly accepted wisdom, moving branes would give rise to particle masses which change in time [3], so that the radion should be very nearly stabilized by the beginning of nucleosynthesis so as not to conflict with observation. One possibility was proposed by Goldberger and Wise, who assumed the existence of a massive bulk scalar field with self-interactions on the branes [4]; see also [5, 6]. By integrating over the fifth dimension one then generates an effective potential for the radion, which has a non-trivial minimum. As the radion field settles into the minimum, the size of the extra dimension gets fixed.

Using a bulk scalar field to stabilize the radion has also been studied in [7]. Other ideas for fixing the size of the fifth dimension include gaugino condensation in a supersymmetric setting [8] and the possibility that the Hubble red shift might damp the radion so that it is almost stabilised [9][4]. A phenomenological mechanism for stabilizing the radion was considered in [3].

The implications of radion stabilization independent of the specifics of the stabilization mechanism were considered in [11, 12]. It was found that a fixed size for the fifth dimension requires a certain non-constant form for the yy-component of the bulk energy-momentum

[4] In [10] the interesting possibility that an asymptotically constant radion emerges from the Einstein equation naturally without the introduction of extra degrees of freedom was considered. Unfortunately the Ansatz used actually precludes any dynamics for the size of the fifth dimension.

tensor. This form turned out to be determined by a constraint that related the yy-component of the stress tensor to its trace. This constraint could be understood in terms of the backreaction of the radion to the inclusion of matter.

Whatever the actual mechanism for stabilizing the radion, it is natural to assume that the fixed size of the extra dimension results from dynamical evolution. In the Goldberger-Wise case, the radion would evolve from its initial state towards the minimum by dissipating the extra energy away. The possibility of the radion stabilizing via decay into Standard Model particles on the TeV brane was considered in [3]. However, it does not seem farfetched to assume that there are other fields in the bulk, which can couple to the radion, whether directly, via the GW field or via some other mechanism, providing an alternative channel for energy dissipation. Such additional fields could *e.g.* be a part of the field content of a supergravity theory in the bulk. The asymptotic final state in the bulk would then involve the radion field with a constant value dictated by the minimum of the effective potential, together with some bulk matter.

If bulk matter exists, at large times one would expect it to be in the state of maximum entropy. This means that e.g. viscous flow should eventually get damped away, and that the bulk matter can be considered an ideal fluid. In that case, stress tensor of the bulk fluid will be spatially homogeneous and isotropic, in particular the yy-component will *not* differ from the other diagonal spatial components, unlike in previous investigations [11, 12, 7, 13, 14, 10, 15, 16].

In section 2 we find the general form of the metric assuming homogeneity and isotropy with respect to the three visible spatial dimensions, one static extra dimension and no matter flow along this extra dimension. These three assumptions impose strong constraints on the metric. We then consider brane matter and show that under the three assumptions of homogeneity and isotropy with respect to the visible spatial dimensions, a static fifth dimension and branes of zero thickness, the only allowed forms of brane matter are a cosmological constant and domain walls moving at the speed of light. This result is in conflict with most literature on RS cosmologies. We comment on the possibility of relaxing the above assumptions in order to be able to include realistic brane matter in the RS model. In section 3 we consider the case of an ideal fluid plus a cosmological constant in the bulk and find the exact cosmological solutions. Curiously, we find that there is only one allowed equation of state for the bulk fluid, $p = \rho$, corresponding to the stiff ideal fluid. We note that it is possible for both branes to have a positive cosmological constant, and one brane may even have a zero cosmological constant, a result discovered in [17] and emphasized in [7]. In section 4 we discuss our results.

2 Static metric

2.1 General form of the static RS metric

The object of our interest is the Einstein equation in 4+1 dimensions,

$$G_{AB} = \kappa^2 T_{AB} \ . \tag{2}$$

In the above, $\kappa^2 = 1/M^3$, where M is the Planck scale in five dimensions. The indices A and B run through time t, three spatial coordinates x^i with infinite range and a compact spatial coordinate y.

Our assumptions regarding the metric and the matter content are as follows:

1. The spacetime is homogeneous and isotropic with respect to the three spatial coordinates x^i.

2. The size of the fifth dimension is constant in time.

3. There is no matter flow in the y-direction.

Assumption 1. The most general metric obeying homogeneity and isotropy with respect to the spatial coordinates x^i is

$$ds^2 = -n(t,y)^2 dt^2 + 2\,c(t,y)\,dt\,dy + b(t,y)^2 dy^2 + \frac{a(t,y)^2}{\left(1+\frac{K}{4}r^2\right)^2}\sum_{i=1}^{3}(dx^i)^2 \ , \tag{3}$$

where $r^2 = \sum_{i=1}^{3}(x^i)^2$ and $6K$ is the constant three-dimensional spatial curvature. For simplicity of notation, we put $K=0$ for the rest of this work. This does not affect our essential results. Any two-dimensional Riemannian manifold is conformally flat, so we can change to conformal coordinates in the (t,y)–subspace. In conformal coordinates, the metric (3) reads

$$ds^2 = -\tilde{b}(\tilde{t},\tilde{y})^2 d\tilde{t}^2 + \tilde{b}(\tilde{t},\tilde{y})^2 d\tilde{y}^2 + \tilde{a}(\tilde{t},\tilde{y})^2 \sum_{i=1}^{3}(dx^i)^2 \ . \tag{4}$$

Assumption 2. When $d\tilde{b}/d\tilde{t} = 0$, the metric (4) reduces to

$$ds^2 = -\tilde{b}(\tilde{y})^2 d\tilde{t}^2 + \tilde{b}(\tilde{y})^2 d\tilde{y}^2 + \tilde{a}(\tilde{t},\tilde{y})^2 \sum_{i=1}^{3}(dx^i)^2 \ . \tag{5}$$

We can now redefine the $\tilde{y}$-coordinate so as to render the metric into the form (we drop the tildes)

$$ds^2 = -f(y)^2 dt^2 + dy^2 + a(t,y)^2 \sum_{i=1}^{3} (dx^i)^2 \ . \tag{6}$$

The above redefinition of the y-coordinate contains a possible problem: if the function $\tilde{b}(\tilde{y})$ has zeros, the transformation from the metric (5) to the metric (6) may be singular. In particular, this singularity may map either or both of the boundaries of the y-coordinate from finite values to infinity, making the fifth dimension non-compact. This issue was discussed in [18], and singularities in the RS scenario have more generally been considered in [19, 17, 20]. We simply assume that the metric is non-singular so that the fifth dimension remains compact.

The Einstein Equation. Let us now introduce an explicit form of the Einstein equation. It is convenient to write the equation in a general gaussian coordinate system, that is, for the metric (3) with $c = 0$. (As mentioned, we also put $K = 0$.) The nontrivial components of the Einstein equation for this metric are

$$\begin{aligned}
G_{tt} &= 3\left[-\frac{n^2}{b^2}\left(\frac{a''}{a} + \frac{a'}{a}\left(\frac{a'}{a} - \frac{b'}{b}\right)\right) + \frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right)\right] = -\kappa^2 n^2 T^t{}_t \\
G_{ii} &= \frac{a^2}{b^2}\left[2\frac{a''}{a} + \frac{n''}{n} + \frac{a'}{a}\left(\frac{a'}{a} + 2\frac{n'}{n}\right) - \frac{b'}{b}\left(\frac{n'}{n} + 2\frac{a'}{a}\right)\right] \\
&\quad - \frac{a^2}{n^2}\left[2\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} - 2\frac{\dot{n}}{n}\right) + \frac{\dot{b}}{b}\left(2\frac{\dot{a}}{a} - \frac{\dot{n}}{n}\right)\right] = \kappa^2 a^2 T^i{}_i \\
G_{yy} &= 3\left[\frac{a'}{a}\left(\frac{a'}{a} + \frac{n'}{n}\right) - \frac{b^2}{n^2}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} - \frac{\dot{n}}{n}\right)\right)\right] = \kappa^2 b^2 T^y{}_y \\
G_{ty} &= 3\left[\frac{n'}{n}\frac{\dot{a}}{a} + \frac{a'}{a}\frac{\dot{b}}{b} - \frac{\dot{a}'}{a}\right] = -\kappa^2 n^2 T^t{}_y \ .
\end{aligned} \tag{7}$$

Dots and primes stand for derivatives with respect to t and y, respectively.

Assumption 3. Inserting $T_{ty} = 0$ into the relevant component of (7) and taking into account $\dot{b} = 0$, we obtain the result

$$\dot{a}(t,y) = A(t) n(t,y) \ , \tag{8}$$

where A is some unknown function. Inserting the result $n(t,y) = f(y)$ from (6) and integrating, we arrive at the following metric:

$$n(t,y) \ = \ f(y)$$

$$
\begin{aligned}
a(t,y) &= a_0(t)f(y)+g(y) \\
b(t,y) &= 1\ .
\end{aligned}
\tag{9}
$$

We emphasize that the line of argumentation leading to the above metric is rather general, and in particular does not depend at all on brane matter or on the specifics of bulk matter.

2.2 Constraints on brane matter

We assume that there are two branes, located at the endpoints of the y-coordinate. The limitation to two branes is not crucial, and more branes could easily be accommodated. We assume that

4. The branes have zero thickness.

According to the above assumption, the stress tensor of brane matter is proportional to a delta function. This assumption is related to the orbifolding of the y-coordinate, which introduces discontinuities in the first y-derivatives of the metric, and delta functions in the second y-derivatives. Since the metric is assumed to be continuous, the only possible delta function contributions to the Einstein tensor come from these second y-derivatives, a'' and n''. This severely restricts the input of the metric to the brane stress tensor (or vice versa). We will now show that if the metric has the form (9), then nothing but cosmological constants and, if $g \neq 0$, domain walls moving at the speed of light can reside on the branes.

The brane part of the Einstein equation (7) reads, in general Gaussian coordinates,

$$
\begin{aligned}
3\frac{1}{b^2}\frac{a''}{a}\bigg|_\delta &= \kappa^2\, T^t{}_t\Big|_{brane} \\
\frac{1}{b^2}\left(2\frac{a''}{a}+\frac{n''}{n}\right)\bigg|_\delta &= \kappa^2\, T^i{}_i\Big|_{brane}\ .
\end{aligned}
\tag{10}
$$

The notations δ and $brane$ refer to the delta function parts of the derivatives and the stress tensor, respectively. With the assumption of spatial homogeneity and isotropy the general form of the brane stress tensor is

$$
T^A{}_B\Big|_{brane} = \sum_{m=1,2}\frac{\delta(y-y_m)}{b(t,y_m)}\mathrm{diag}(-\rho_m(t),p_m(t),p_m(t),p_m(t),0)\ , \tag{11}
$$

where the index m enumerates the branes, and we take $y_2 > y_1$. On the other hand, the delta function part of the metric is related to the jumps of the first derivatives of the metric, which can in turn be expressed in terms of the continuous part of the metric [21]:

$$
\begin{aligned}
a''|_\delta &= \sum_{m=1,2} \delta(y-y_m)[a'] \\
&= \sum_{m=1,2} \delta(y-y_m)(-1)^{m+1} 2\, a'_c \ ,
\end{aligned} \tag{12}
$$

where $[a']$ is the discontinuity of a', $[a'(y)] := \lim_{\varepsilon\to 0}(a'(y+\varepsilon) - a'(y-\varepsilon))$, and a'_c is the continuous part of a'. An identical relation holds for n. Putting together (10), (11) and (12), we have

$$
\pm\frac{2}{b}\frac{a'_c}{a}\bigg|_{y=y_m} = -\frac{\kappa^2}{3}\rho_m \tag{13}
$$

$$
\pm\frac{2}{b}\frac{n'_c}{n}\bigg|_{y=y_m} = \frac{\kappa^2}{3}(3p_m + 2\rho_m) \ . \tag{14}
$$

Now, consider the metric (9). If $g(y) = 0$, so that the metric factorizes, the left hand sides of (13) and (14) become equal and independent of time. Then the only possible equation of state is that of a cosmological constant, $p_m = -\rho_m$. Giving up factorizability ($g \neq 0$) does not allow much more freedom. In that case (13) allows ρ_m to be time-dependent, but because the *lhs* of (14) is time-independent, the time-dependent part of $3p_m + 2\rho_m$ must cancel. That corresponds to a time-dependent density of two-dimensional domain walls moving at the speed of light on the brane. So the most general equation of state is

$$
\rho_m(t) = \rho_{m(wall)}(t) + \rho_{m(vacuum)} = -\frac{3}{2}p_{m(wall)}(t) - p_{m(vacuum)} \ . \tag{15}
$$

The equation (13) leads to the same conclusion if we apply the conservation law of the brane stress tensor.

The assumption of no matter flow along the fifth dimension is actually not needed in the above proof: in conformal coordinates n and b are equal, so that $\dot{b} = 0$ automatically implies $\dot{n} = 0$, leading to the above result. We conclude that homogeneity and isotropy, a static fifth dimension and branes with zero thickness (the orbifolding) together forbid any realistic brane matter.

We emphasize that the above result is in conflict with much of the existing literature on RS cosmology. Several RS-type solutions satisfying our four assumptions and allegedly containing brane matter with arbitrary equations of state have been presented [3, 11, 12, 13, 21, 22, 23, 15], to mention a few. We have explicitly checked some of these solutions and found that they in fact require $\dot{\rho}_m = 0$. But then the conservation law of the stress tensor implies that $\rho_m = -p_m$ – so the brane matter must correspond to a cosmological constant.

Since it should certainly be possible to include matter with an arbitrary equation of state on the branes and the assumptions 1, 2 and 4 are the only ingredients of our proof, at least one of these assumptions must be relaxed.

The visible universe is known to be homogeneous and isotropic at large scales to a high degree of accuracy, as attested to by the cosmic microwave background, so giving up assumption 1 seems rather unattractive.

If the fifth dimension has an orbifold structure, the branes automatically have zero thickness. If one gives up the orbifolding and considers branes with finite thickness, the whole range of derivatives in the Einstein tensor can contribute to the brane stress tensor. Then a strict constraint like (15) does not seem likely to emerge, and realistic brane matter can possibly be included. Branes with finite thickness have been considered in [11, 12, 19].

Time-dependence of b. Perhaps the most viable option for including brane matter is to allow for a time-dependent b. It has been argued that time dependence in b leads to time-varying particle masses [3]. Then any change in b must be negligible by the time of nucleosynthesis so as not to contradict observation. As an aside, we note that these arguments have depended on a specific form of the metric, and that it is not obvious that a time-dependent b would in general lead to such time-dependence of n and a as to affect particle masses.

However, in this paper we assume that b does need to be approximately fixed, whether it is done to avoid inducing time-dependence in particle masses or for some other reason. (In the next section, we will give an argument to support the idea that the RS scenario requires an at least approximately time-independent b.) We may still consider the possibility that b is almost stabilized but varies in time slowly enough not to conflict with observation, either asymptotically approaching a constant value or oscillating about a minimum. A b varying slowly with time might possibly allow for the inclusion of realistic brane matter. The time-independent b can then be considered an approximation, and perhaps an asymptotic limit, of this scenario. Treating time-dependent brane matter as a perturbation against a background of bulk matter (including possibly a cosmological constant) and brane cosmological constants is supported by the fact that energy density of the universe from the time of nucleosynthesis onwards is quite small in natural units, even if the scale of the five-dimensional gravitation is in the TeV range. Also, it seems in general more plausible that dynamical evolution would lead to a b with weak time-dependence rather than a completely fixed b.

In what follows, we take b to be constant, and assume that a weak time-dependence as well as realistic brane matter can be included as a perturbation. This line of thought has been pursued in [3, 24].

2.3 Motivation for a static fifth dimension

An integral assumption in most papers on the RS model, including this one, is the assumption of a static fifth dimension. Since it is does not seem obvious that a non-static fifth dimension would in general lead to conflict with observation, we now present a sketch to motivate the assumption $\dot{b} = 0$.

The RS model was first envisaged as a solution to the hierarchy problem: the (four-dimensional) length scale increases as one moves from one brane to the other, which induces a change in the mass scales between the branes. Strictly speaking, the solution of the hierarchy problem only requires that the functions $n(t, y)$ and $a(t, y)$ are such that we have (preferably without introducing large numbers)

$$\begin{aligned} \frac{n(t, y_1)}{n(t, y_2)} &= N \\ \frac{a(t, y_1)}{a(t, y_2)} &= Nh(t) \ , \end{aligned} \tag{16}$$

where y_1, y_2 are the two brane positions, $N \sim 10^{16}$ is the ratio of the Planck and TeV scales and $h(t)$ is some function of time (to allow for the possibility of different cosmological expansion factors on the two branes). This is a rather weak constraint, since it makes no reference to behaviour away from the branes. However, we can obtain a stronger and therefore a more useful constraint by making the additional Ansatz that the condition (16) holds not only for the particular value $y = y_1$, but for *all* values of y, with the value N replaced by some function $f(y)$. In other words, the Ansatz says that the Einstein equation implies no preferred brane positions. This Ansatz implies that

$$\begin{aligned} n(t, y) &= n_0(t) f(y) \\ a(t, y) &= a_0(t) f(y) \\ b(t, y) &= b(t, y) \ . \end{aligned} \tag{17}$$

Substituting (17) into the ty-component of the Einstein equation (7) and assuming $T_{ty} = 0$, we obtain the condition

$$f' \dot{b} = 0 \ . \tag{18}$$

For a nontrivial warp factor, the above equation can only be satisfied if $\dot{b} = 0$. We then redefine the coordinates t and y to set the functions $n_0(t)$ and $b(y)$ to unity, so that the metric reads

$$\begin{aligned} n(t, y) &= f(y) \\ a(t, y) &= a_0(t) f(y) \\ b(t, y) &= 1 \ . \end{aligned} \tag{19}$$

Thus, the factorisable metric (19) and the condition $\dot{b} = 0$ can be motivated by requiring Randall-Sundrum-type solutions. If the branes contain a perturbatively small matter density which induces weak time-dependence in b, then we might expect to approximately recover the factorizable metric (19), as noted in [23].

3 The Einstein Equation with Ideal Fluid

3.1 The stress tensor

After general investigations of the metric and brane matter, we now proceed to study the particular case of a bulk with ideal fluid and a cosmological constant.

Bulk matter, either in the form of scalar fields or a cosmological fluid (which can sometimes be interpreted as a scalar field, and vice versa) has been considered in many papers. However, the scalar field studies, of which we mention only a few, have concentrated on fixing the size of the fifth dimension [4, 5, 6], inflation [18], the cosmological constant problem [20], singularities and the adS/CFT-correspondence [19], or on more general aspects of the formalism [6], not on obtaining cosmological solutions. In papers of a more cosmological nature, often only the yy-component of the bulk stress tensor has been allowed to deviate from a cosmological constant [11, 12, 7, 13, 14], and in any case the yy-component has been taken to be different from the $x^i x^i$-components [10, 15, 16][5], so that the bulk matter cannot be interpreted as an ideal fluid. Furthermore, complete and explicit cosmological solutions for the case of a bulk stress tensor with non-trivial tt- and $x^i x^i$-components are rarely presented; [24] mentions one in passing. We will now present one such solution.

For the moment, we do not make any assumptions about the metric. We take the branes and the bulk to contain some ideal fluids of unspecified nature. It is clearest to introduce a local orthonormal frame to find the form of the stress tensor. We introduce coordinates $(\hat{x}^{\hat{A}}) = (\hat{t}, \hat{x}^{\hat{i}}, \hat{y})$ such that locally the five-dimensional line element takes the form of the 5-d Minkowski metric,

$$ds^2 = -d\hat{t}^2 + \sum_{\hat{i}=1}^{3} (d\hat{x}^{\hat{i}})^2 + d\hat{y}^2 \ . \tag{20}$$

In the local orthonormal frame, the stress tensor for brane (bulk) ideal fluids must be homogeneous and isotropic in the three (four) spatial dimensions on the branes (in the

[5][16] is particularly interesting and has certain results which are close to ours despite the somewhat different setting.

bulk). We thus have

$$T^{\hat{A}}{}_{\hat{B}} = \sum_{m=1,2} \delta(\hat{y} - \hat{y}_m)\mathrm{diag}(-\rho_m, p_m, p_m, p_m, 0) + \mathrm{diag}(-\rho - \Lambda, p - \Lambda, p - \Lambda, p - \Lambda, p - \Lambda) \ . \tag{21}$$

In the above, ρ_m, p_m are the energy densities and pressures of ideal fluid on the two branes located at

$$\hat{y}_1 = \int_0^{y_1} dy' b(t, y') \ ; \ \hat{y}_2 = \int_0^{y_2} dy' b(t, y') \ , \tag{22}$$

ρ, p are the energy density and pressure of the bulk ideal fluid and Λ is the bulk cosmological constant which we have for convenience separated out. In particular, note that in the local orthonormal frame the pressure of an ideal fluid in the $\hat{y}$ direction is *equal* to the pressure in the $\hat{x}^{\hat{i}}$ directions. We now introduce the assumption of homogeneity and isotropy in the directions parallel to the brane. Then the pressures and energy densities cannot depend on the coordinates $\hat{x}^{\hat{i}}$, only on the time $\hat{t}$ and, in the case of the bulk fluid, the perpendicular direction $\hat{y}$. Thus, expressing the stress tensor in the original coordinates, we have

$$T^{A}{}_{B} = \sum_{m=1,2} \frac{\delta(y - y_m)}{b(t, y_m)}\mathrm{diag}(-\rho_m(t), p_m(t), p_m(t), p_m(t), 0) + \mathrm{diag}(-\rho(t, y) - \Lambda, p(t, y) - \Lambda, p(t, y) - \Lambda, p(t, y) - \Lambda, p(t, y) - \Lambda) \ . \tag{23}$$

The bulk ideal fluid is assumed to satisfy a linear equation of state,

$$p(t, y) = w\rho(t, y) \ . \tag{24}$$

We could for generality write the bulk ideal fluid as a sum of components with different w, but since it turns out that w can take only one value, we prefer not to clutter the notation. Also, one could in principle allow the coefficient w to depend on t and y, describing a time- and coordinate-dependent (that is, interacting) mixture of ideal fluids. However, in this paper we shall assume that w is constant.

3.2 Static fifth dimension equals factorization

We now simplify the metric (9) by showing that when the bulk contains ideal fluid, we have $g = 0$ and the metric factorizes. The proof utilises the conservation law of the stress tensor,

$$D_A T^{A}{}_{B} = 0 \ . \tag{25}$$

The above conservation law implies that matter on the branes and in the bulk satisfies the following equations (we take gaussian coordinates),

$$\begin{aligned}
\dot{\rho}_m + 3(\rho_m + p_m)\frac{\dot{a}}{a}\Big|_{y=y_m} &= 0 \qquad (26)\\
\dot{\rho} + (\rho + p)\left(3\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right) &= 0 \qquad (27)\\
p' + (\rho + p)\frac{n'}{n} &= 0\ . \qquad (28)
\end{aligned}$$

Interestingly, it follows from (28) that it is not possible to have $p = 0$ unless also $\rho = 0$ (assuming that the warp factor is non-trivial). So, the conservation law of the stress tensor restricts the possible equations of state for the bulk ideal fluid. We will shortly see that the full Einstein equation as a matter of fact permits only one particular equation of state (in addition to the equation of state of a cosmological constant), that of the stiff ideal fluid.

Substituting the equation of state (24), we can integrate the equations (27) and (28) for the bulk fluid to obtain

$$\begin{aligned}
\rho(t,y) &= B(y)\, a(t,y)^{-3(1+w)}\, b(t,y)^{-(1+w)}\\
\rho(t,y) &= C(t)\, n(t,y)^{-\frac{1+w}{w}}\ , \qquad (29)
\end{aligned}$$

where B and C are some unknown functions. Introducing the assumption $\dot{b} = 0$, the equations (29) imply the following relation between the metric functions:

$$n(t,y) = B(y)^{-\frac{w}{1+w}} C(t)^{\frac{w}{1+w}} a(t,y)^{3w}\ . \qquad (30)$$

Comparing with (9), we see that $g = 0$ or $\dot{a}_0 = 0$. As we are interested in cosmological solutions, the possibility $\dot{a}_0 = 0$ is excluded and we have $g = 0$. We arrive at the factorizable metric (19):

$$\begin{aligned}
n(t,y) &= f(y)\\
a(t,y) &= a_0(t) f(y)\ . \qquad (31)
\end{aligned}$$

We note that since the metric is factorizable, matter on the branes reduces to cosmological constants, according to (13).

3.3 Solution of the Einstein equation

We have shown that in the case of ideal fluid in the bulk, our four assumptions lead to the factorizable metric (31). We now proceed to solve the Einstein equation with this metric.

Since the Einstein equation is local and the metric is assumed to be continuous, the branes contribute only to boundary conditions. Hence we can ignore the brane contribution to the stress tensor in local bulk calculations. With the factorizable metric (31), the ty-component of the Einstein equation is satisfied trivially. The remaining components of (7) with the the stress tensor (23) and the equation of state (24) read, away from the branes:

$$\begin{aligned}
\frac{f''}{f}+\frac{f'^2}{f^2}-\frac{1}{f^2}\left(\frac{\dot{a}_0}{a_0}\right)^2 &= -\hat{\rho}-\hat{\Lambda} \\
\frac{f''}{f}+\frac{f'^2}{f^2}-\frac{1}{f^2}\left(\frac{2}{3}\frac{\ddot{a}_0}{a_0}+\frac{1}{3}\left(\frac{\dot{a}_0}{a_0}\right)^2\right) &= w\hat{\rho}-\hat{\Lambda} \\
2\frac{f'^2}{f^2}-\frac{1}{f^2}\left(\frac{\ddot{a}_0}{a_0}+\left(\frac{\dot{a}_0}{a_0}\right)^2\right) &= w\hat{\rho}-\hat{\Lambda}\ ,
\end{aligned} \tag{32}$$

where

$$\hat{\rho}:=\frac{\kappa^2}{3}\rho\ ,\ \ \hat{\Lambda}:=\frac{\kappa^2}{3}\Lambda\ . \tag{33}$$

Taking linear combinations of the above equations, we obtain the following equivalent set:

$$\hat{\rho} = \frac{2}{3(w+1)}\left(\left(\frac{\dot{a}_0}{a_0}\right)^2-\frac{\ddot{a}_0}{a_0}\right)f^{-2} \tag{34}$$

$$\frac{f''}{f} = \frac{w-1}{4}\hat{\rho}-\frac{\hat{\Lambda}}{2} \tag{35}$$

$$f''f-(f')^2 = -\frac{1}{3}\frac{\ddot{a}_0}{a_0}-\frac{2}{3}\left(\frac{\dot{a}_0}{a_0}\right)^2\equiv-\frac{\mathrm{C}}{9}\ . \tag{36}$$

In the last equation we have introduced a constant C. The *l.h.s.* of (35) is independent of t, while from (34) we see that $\hat{\rho}$ depends[6] on t. Thus, we obtain the result $w=1$. In other words, the only allowed equation state for the bulk ideal fluid is

$$p=\rho\ . \tag{37}$$

This result does not depend on the choice $K=0$. We postpone the discussion of the properties of an ideal fluid with this equation of state to section 3, and proceed to find the corresponding cosmological solutions.

With $w=1$, the equations (34) to (36) read

$$\hat{\rho} = \frac{1}{3}\left(\left(\frac{\dot{a}_0}{a_0}\right)^2-\frac{\ddot{a}_0}{a_0}\right)f^{-2} \tag{38}$$

[6]The possibility $\frac{\dot{a}_0^2}{a_0^2}-\frac{\ddot{a}_0}{a_0}=$ constant would reduce the bulk fluid to a cosmological constant.

$$\frac{f''}{f} = -\frac{\hat{\Lambda}}{2} \tag{39}$$

$$f''f - (f')^2 = -\frac{\mathrm{C}}{9} \tag{40}$$

$$\frac{\ddot{a}_0}{a_0} + 2\left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{\mathrm{C}}{3} \, . \tag{41}$$

For $K \neq 0$, the time dependence of $\hat{\rho}$ and the equation for a_0 would change, but the equations for the warp factor would remain unchanged. Also note that the bulk cosmological constant contributes only to the warp factor, not to the cosmological expansion factor a_0. It is interesting to see that the bulk fluid accumulates at the brane where f has its minimum value, in other words at the TeV brane, a result noted by [10].

Comparison with the FRW model. It is interesting to compare the equations (38) and (41) with the corresponding equations for the $K = 0$ Friedmann-Robertson-Walker model. We take the same matter content for the FRW model: an ideal fluid with the equation of state $p_{(3)} = \rho_{(3)}$ and a cosmological constant $\Lambda_{(3)}$ (we use the subscript 3 as a reminder that there are only three spatial dimensions). The Einstein equation for the FRW model with this matter content reads

$$\hat{\rho}_{(3)} = \frac{1}{3}\left(\left(\frac{\dot{a}}{a}\right)^2 - \frac{\ddot{a}}{a}\right) \tag{42}$$

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 = 3\hat{\Lambda}_{(3)} \, . \tag{43}$$

Comparison of (38) and (41) with (42) and (43) shows that a_0 in the RS case is exactly the same as a in the FRW case, with C playing the role of a cosmological constant, $\mathrm{C} = 9\hat{\Lambda}_{(3)}$, and that $\hat{\rho}$ equals $\hat{\rho}_{(3)}$, save for the factor f^{-2}. These results hold also for $K \neq 0$. So, bulk ideal fluid in the RS model behaves exactly like ideal fluid in the FRW model, modulo the warp factor. However, we will shortly see that in the RS case only the sign of C has any physical meaning, in contrast to the FRW case, where the magnitude of $\hat{\Lambda}_{(3)}$ sets the timescale of the universe.

We analyze the cases $\mathrm{C} = 0$ and $\mathrm{C} \neq 0$ separately.

$\mathbf{C = 0}$. The equations (38) to (41) now read

$$\hat{\rho} = \frac{1}{3}\left(\left(\frac{\dot{a}_0}{a_0}\right)^2 - \frac{\ddot{a}_0}{a_0}\right) f^{-2} \tag{44}$$

$$\frac{f''}{f} = -\frac{\hat{\Lambda}}{2} \tag{45}$$

$$
\begin{aligned}
f''f - (f')^2 &= 0 && (46)\\
\frac{\ddot{a}_0}{a_0} + 2\left(\frac{\dot{a}_0}{a_0}\right)^2 &= 0\ . && (47)
\end{aligned}
$$

The above equations have the following unique[7] solution

$$
\begin{aligned}
a_0(t) &= \left(\frac{t}{\tau}\right)^{1/3} && (48)\\
f(y) &= f_0 e^{\pm y\sqrt{-\hat{\Lambda}/2}} && (49)\\
\rho(t,y) &= \frac{1}{3\kappa^2}\frac{1}{t^2} f(y)^{-2}\ . && (50)
\end{aligned}
$$

We have normalized by $a_0(0) = 0$. The bulk cosmological constant has to be negative, $\hat{\Lambda} < 0$.

The solution contains two free parameters, τ and f_0. A choice of the time parameter τ corresponds to choosing the unit of time, so that τ has no physical meaning, as in the FRW case. A choice of the warp parameter f_0 corresponds to choosing the origin of the y-coordinate. The metric and the energy density are invariant under the scaling $t \to \lambda t$, $x^i \to \lambda x^i$, $f_0 \to \lambda^{-1} f_0$, so that f_0, or the placement of the origin, has no physical meaning.

The $\mathrm{C} = 0$ solution contains no physical degrees of freedom other than the value of the bulk cosmological constant: the metric and the bulk energy density (and, as we will see later, the brane cosmological constants) are fixed once the bulk cosmological constant is specified. This suggests that the solution is unstable, possibly collapsing to one of the $\mathrm{C} \neq 0$ solutions when perturbed.

$\mathbf{C} \neq \mathbf{0}$. In this case the equations (38) to (41) read

$$
\begin{aligned}
\hat{\rho} &= \frac{1}{3}\left(\left(\frac{\dot{a}_0}{a_0}\right)^2 - \frac{\ddot{a}_0}{a_0}\right) f^{-2} && (51)\\
\frac{f''}{f} &= -\frac{\hat{\Lambda}}{2} && (52)\\
f''f - (f')^2 &= -\frac{\mathrm{C}}{9} && (53)\\
\frac{\ddot{a}_0}{a_0} + 2\left(\frac{\dot{a}_0}{a_0}\right)^2 &= \frac{\mathrm{C}}{3}\ . && (54)
\end{aligned}
$$

The above equations have the solution[8]

$$
a_0(t) = a_1 \begin{cases} \sin^{1/3}(\sqrt{|\mathrm{C}|}\ t) & \mathrm{C} < 0 \\ \sinh^{1/3}(\sqrt{\mathrm{C}}\ t) & \mathrm{C} > 0 \end{cases} \qquad (55)
$$

[7]Apart from the trivial solution $\dot{a}_0 = 0$, which would lead to $\rho = 0$ and the original RS model.

[8]The equation for a_0 allows de Sitter and anti-de Sitter solutions when $\mathrm{C} > 0$, but these would again give $\rho = 0$. These solutions were first presented in [25, 17].

$$
\begin{aligned}
f(y) &= f_1 e^{y\sqrt{-\hat{\Lambda}/2}} + \frac{\mathrm{C}}{18\hat{\Lambda} f_1} e^{-y\sqrt{-\hat{\Lambda}/2}} && (56)\\
\rho(t,y) &= \frac{|\mathrm{C}|}{3\kappa^2} f(y)^{-2} \begin{cases} \sin^{-2}(\sqrt{|\mathrm{C}|}\ t) & \mathrm{C}<0 \\ \sinh^{-2}(\sqrt{\mathrm{C}}\ t) & \mathrm{C}>0 \end{cases} . && (57)
\end{aligned}
$$

We have again used the normalization $a_0(0) = 0$. The equations for f would allow a positive (zero) bulk cosmological constant, resulting in trigonometric functions (a linear function), but in order to solve the hierarchy problem we take $\hat{\Lambda} < 0$.

As in the previous case, the parameter a_1 has no physical meaning, and can be set to unity, while f_1 corresponds to the choice of the origin of y. Rewriting f_1 in terms of a coordinate value y_0, we have

$$
\begin{aligned}
a_0(t) &= \begin{cases} \sin^{1/3}(\sqrt{|\mathrm{C}|}\ t) & \mathrm{C}<0 \\ \sinh^{1/3}(\sqrt{\mathrm{C}}\ t) & \mathrm{C}>0 \end{cases} && (58)\\
f(y) &= \sqrt{\frac{2|\mathrm{C}|}{9|\hat{\Lambda}|}} \begin{cases} \cosh\left(\sqrt{|\hat{\Lambda}|/2}\ (y-y_0)\right) & \mathrm{C}<0 \\ \sinh\left(\sqrt{|\hat{\Lambda}|/2}\ (y-y_0)\right) & \mathrm{C}>0 \end{cases} && (59)\\
\rho(t,y) &= \frac{|\Lambda|}{2} \begin{cases} \cosh^{-2}\left(\sqrt{|\hat{\Lambda}|/2}\ (y-y_0)\right)\ \sin^{-2}(\sqrt{|\mathrm{C}|}\ t) & \mathrm{C}<0 \\ \sinh^{-2}\left(\sqrt{|\hat{\Lambda}|/2}\ (y-y_0)\right)\ \sinh^{-2}(\sqrt{\mathrm{C}}\ t) & \mathrm{C}>0 \end{cases} . && (60)
\end{aligned}
$$

We see that the metric and the energy density are invariant under the scaling $t \to \lambda t$, $x^i \to \lambda x^i$, $\mathrm{C} \to \lambda^{-2}\mathrm{C}$. In other words, only the sign of C has any physical meaning, the magnitude is irrelevant. In particular, C does not introduce a new mass scale into the model.

Unlike in the case $\mathrm{C} = 0$ (and the original RS proposal) the model is *not* invariant under translations of the y-coordinate, so that the choice of origin of y has a physical meaning. The translational invariance is broken by the y-dependence in the bulk matter: y_0 is not a free parameter but is set by the bulk energy density, as we see from (60). There are two physical degrees of freedom: the value of the bulk cosmological constant and the bulk energy density. (In addition, there are of course the brane cosmological constants, which, as we will see shortly, are also free parameters.)

In the case $\mathrm{C} > 0$ the function f has a zero at $y = y_0$. The bulk energy density and the scalar curvature diverge when $f = 0$, so the singularity at $y = y_0$ is physical, not an artifact of the coordinate system we have chosen. In what follows, we simply avoid the singularity by constraining the value y_0 not to lie between the brane positions in the case $\mathrm{C} > 0$.

3.4 Branes and fine-tuning

Having solved the Einstein equation in the bulk, we now turn to the branes, which, as noted earlier, provide boundary conditions.

Brane cosmological constants. The factorizable metric implies that the branes contain only cosmological constants. We switch to the notation $\rho_m = -p_m =: \Lambda_m$. Inserting the brane contribution to the stress tensor from (23) into the Einstein equation (7), we get

$$\left.\frac{[f']}{f}\right|_{y=y_m} \;=\; -\hat{\Lambda}_m \;, \tag{61}$$

where y_1, y_2 ($y_2 > y_1$) are the orbifold fixed points, where the branes are located[9]. As before, the notation $[f']$ refers to the discontinuity of f'. Writing the discontinuity in terms of the continuous part f'_c, we have

$$\begin{aligned} \left.2\frac{f'_c}{f}\right|_{y=y_1} &= -\hat{\Lambda}_1 \\ \left.-2\frac{f'_c}{f}\right|_{y=y_2} &= -\hat{\Lambda}_2 \;. \end{aligned} \tag{62}$$

For C $= 0$ (and the original RS proposal) we have $f(y) = f_0 e^{-|y|\sqrt{|\hat{\Lambda}|/2}}$. Then (62) gives the result $\hat{\Lambda}_1 = -\hat{\Lambda}_2 = \sqrt{2|\hat{\Lambda}|}$, an unfortunate fine-tuning. However, for C < 0 we have instead

$$\begin{aligned} \hat{\Lambda}_1 &= -\sqrt{2|\hat{\Lambda}|}\tanh(\sqrt{|\hat{\Lambda}|/2}\;(y_1 - y_0)) \\ \hat{\Lambda}_2 &= \sqrt{2|\hat{\Lambda}|}\tanh(\sqrt{|\hat{\Lambda}|/2}\;(y_2 - y_0)) \;; \end{aligned} \tag{63}$$

for C > 0 the hyperbolic tangent is replaced by a hyperbolic cotangent[10]. The above equations for the case C < 0 were found in [7] in a slightly different setting. The values of hyperbolic tangent lie in the interval from –1 to +1 (excluding ± 1), and the values of hyperbolic cotangent stretch from minus infinity to plus infinity, excluding the interval from –1 to +1. Thus, the ratio of the brane cosmological constants to the square-root of the bulk cosmological constant determines the value of C:

$$\mathrm{C} \;<\; 0 \qquad \text{for } \frac{|\hat{\Lambda}_m|}{\sqrt{2|\hat{\Lambda}|}} < 1$$

[9]In the case C > 0 we demand $y_0 < y_1$ or $y_0 > y_2$ to avoid having a singularity.
[10]Note that either set of equations fixes $y_2 - y_1$, the size of the fifth dimension.

$$
\begin{aligned}
\mathrm{C} &= 0 \qquad \text{for} \quad \frac{|\hat{\Lambda}_m|}{\sqrt{2|\hat{\Lambda}|}} = 1 \\
\mathrm{C} &> 0 \qquad \text{for} \quad \frac{|\hat{\Lambda}_m|}{\sqrt{2|\hat{\Lambda}|}} > 1 \ .
\end{aligned} \tag{64}
$$

In the present model, the Einstein equation implies no fine-tuning of parameters, unlike in the original RS proposal. For a given value of the bulk cosmological constant $\hat{\Lambda}$, the absolute values of both brane cosmological constants have to be smaller than, bigger than or equal to $\sqrt{2|\hat{\Lambda}|}$, but are otherwise unrestricted by the Einstein equation. With two equations to satisfy and two constants, $y_1 - y_0$ and $y_2 - y_0$, at our disposal, no fine-tuning is needed to obtain a solution to (62).

The signs of the brane cosmological constants are opposite for $\mathrm{C} = 0$ and $\mathrm{C} > 0$. This can also be the case for $\mathrm{C} < 0$. However, for $\mathrm{C} < 0$ it is also possible for both brane cosmological constants to be positive: this requires the value of y_0 to lie between the brane positions, $y_1 < y_0 < y_2$. Furthermore, we have the interesting possibility of a *zero* cosmological constant on one brane and a positive cosmological constant on the other: this requires one of the branes to be placed at the point where f has its minimum, $y_0 = y_1$ or $y_0 = y_2$. (It is not possible to have two branes with non-positive cosmological constants.) Thus, there is no need for negative energy densities on the branes, essentially because the warp factor is not a monotonical function, but has a minimum. This was first noticed in [17] and further emphasized in [7]. Of course, the bulk cosmological constant still has to be negative to obtain an exponential warp factor.

The hierarchy problem. The solution to the hierarchy problem introduces a new equation, namely

$$
\frac{f(y_1)}{f(y_2)} \sim 10^{\pm 16} \ . \tag{65}
$$

For $\mathrm{C} < 0$, the above equation reads

$$
\frac{\cosh(\sqrt{|\hat{\Lambda}|/2}\ (y_1 - y_0))}{\cosh(\sqrt{|\hat{\Lambda}|/2}\ (y_2 - y_0))} \sim 10^{\pm 16} \ ; \tag{66}
$$

for $\mathrm{C} > 0$ the hyperbolic cosine is replaced by a hyperbolic sine. Combining (66) with (63), we can write the hierarchy condition (65) as the following equation, valid for all values of C:

$$
\frac{\hat{\Lambda}_2^2}{2|\hat{\Lambda}|} - 1 \sim 10^{\pm 32} \left(\frac{\hat{\Lambda}_2^2}{2|\hat{\Lambda}|} - 1 \right) \ . \tag{67}
$$

We see that the only way to avoid introducing unnaturally large numbers is to fine-tune the cosmological constants, $\hat{\Lambda}_1^2 = \hat{\Lambda}_2^2 = 2|\hat{\Lambda}|$, corresponding to $\mathrm{C} = 0$.

The above result is not dependent on the presence of bulk ideal fluid: if we put $\rho = 0$, our solutions disappear, but we get new solutions with the same problems. For $\mathrm{C} = 0$ we get a constant a_0 (the original RS solution) and for $\mathrm{C} > 0$ we get de Sitter and anti-de Sitter solutions, found in [25, 17]; the case $\mathrm{C} < 0$ becomes disallowed. The warp factor is unchanged by the absence of bulk ideal fluid (except that y_0 becomes a free parameter), so the condition (67) is also unchanged. This means that the fine-tuning problem is *not* inherent to the Einstein equation. Specifically, it is not due to the condition $\dot{b} = 0$, as sometimes claimed [3]. It is rather a rephrasing of the hierarchy problem. In this sense the RS model does not provide quite a satisfactory solution to the hierarchy problem.

4 Discussion

As is well known, the RS solution is quite precarious; bulk and brane sources for gravity must be chosen very carefully for the RS solution to emerge from the Einstein equation. However, it appears that the sources are more severely constrained than what is often thought in the literature. It turns out to be very difficult to introduce realistic matter into the bulk or branes. The latter feature in particular creates problems for trying to interpret our universe as a brane world.

In Section 2 we found that under minimal assumptions on the RS model, a stabilized radion and zero thickness branes, one cannot introduce realistic matter[11] onto the branes. The only allowed equation of state, other than that of a cosmological constant, corresponds to a fluid of two dimensional domain walls moving at the speed of light. Even that requires giving up factorizability for the bulk metric. Our result is therefore in direct contradiction with many claims for brane matter solutions in the literature. We have explicitly checked some of the earlier solutions and found that in fact they satisfy the Einstein equation and the conservation law for the stress tensor only when the brane matter corresponds to a cosmological constant.

The dynamical stabilization of the radion degree of freedom can also be expected to be highly nontrivial. Nevertheless, unless for some reason the universe started out with all the degrees of freedom in the ground state, some dynamics should be expected. In the context of cosmology, a natural initial condition for the radion could be any value compatible with the uncertainty principle, as in chaotic inflation. The radion potential energy must then be released in some way. It is conceivable that the extra radion energy

[11] This means homogenous and isotropic matter with an equation of state corresponding to a combination of dust or radiation.

is dissipated into purely gravitational degrees of freedom, but there is no guarantee that a RS-type solution would result; this remains to be studied. The other possibility, radion decaying into bulk degrees of freedom, assumed in this paper, is possible but highly constrained, as we have shown. The only admissable equation of state for ideal fluid in the bulk was found to be $p = \rho$, representing the so called stiff ideal fluid. Here "stiff" reflects the fact that the velocity of sound in the fluid is equal to the velocity of light. Concretely, such a fluid corresponds to a classical free massless, coherent scalar field (not to be confused with massless radiation), something that was considered in a static setting in [7]. Further, the massless bulk scalar field should not be confused with the massive Goldberger-Wise bulk scalar field. Whether such a field could be coupled to the radion is beyond the scope of the present study.

Given the no-go flavor of our results, it would be interesting to investigate if other equations of state for the brane or bulk ideal fluids could be allowed by some modification of the problem. In the case of the bulk, a natural guess would be to start with a different Ansatz for the bulk geometry. This corresponds to a different assumption on the spacetime symmetries. For example, if the bulk geometry would correspond to an adS black hole, one might expect that the equation of state for massless radiation would become allowed.

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